A New Class of Inhomogeneous String Cosmological Models in General Relativity

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Abstract

A new class of solutions of Einstein field equations has been investigated for inhomogeneous cylindrically symmetric space-time with string source. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. Certain physical and geometric properties of the models are also discussed.

Keywords : String, Inhomogeneous universe, Cylindrical symmetry
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1 Introduction

In recent years, there has been considerable interest in string cosmology because cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich et al., 1975; Kibble, 1976, 1980; Everett, 1981; Vilenkin, 1981). Moreover, the investigation of cosmic strings and their physical processes near such strings has received wide attention because it is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies (Zel'dovich, 1980; Vilenkin, 1981). These cosmic strings have stress

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energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein's equations.

The general treatment of strings was initiated by Letelier (1979, 1983) and Stachel (1980). Letelier (1979) obtained the general solution of Einstein's field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier (1983) also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Benerjee et al. (1990) have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a\beta$ between metric potential where $\alpha = \alpha(t)$ and $\beta = \beta(t)$ and a is constant. Exact solutions of string cosmology for Bianchi type-II, $-VI_0$, -VIII and -IX space-times have been studied by Krori et al. (1990) and Wang (2003). Wang (2004, 2005, 2006) has investigated bulk viscous string cosmological models in different space-times. Bali et al. (2001, 2003, 2005, 2006, 2007) have obtained Bianchi type-I, -III, -V and type-IX string cosmological models in general relativity. The string cosmological models with a magnetic field are discussed by Chakraborty (1991), Tikekar and Patel (1992, 1994), Patel and Maharaj (1996). Ram and Singh (1995) obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh (1980). Singh and Singh (1999) investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh (1995) has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland (2000) have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Yavuz et al. (2005) have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Recently Kaluza-Klein cosmological solutions are obtained by Yilmaz (2006) for quark matter attached to the string cloud in the context of general relativity.

Cylindrically symmetric space-time play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically symmetric cosmological models have significant contribution in understanding some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali and Tyagi (1989) and Pradhan et al. (2001, 2006) have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field. Barrow and Kunze (1997, 1998) found a wide class of exact cylindrically symmetric flat and open inhomogeneous string universes. In their solutions all physical quantities depend on at most one space coordinate and the time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure equal to energy density.

Recently Baysal et al. (2001), Kilinc and Yavuz (1996) have investigated some string cosmological models in cylindrically symmetric inhomogeneous universe. In this paper, we have revisited their solutions and obtained a new class of solutions. Here, we extend our understanding of inhomogeneous string cosmologies by investigating the simple models of non-linear cylindrically symmetric inhomogeneities outlined above. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in two different cases. Finally, the results are discussed in Section 4. The solutions obtained in this paper are new and different from the other author's solutions.

2 The Metric and Field Equations

We consider the Bianchi Type I metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B and C are functions of x and t. The Einstein's field equations for a cloud of strings read as (Letelier, 1983)

$$G_i^j \equiv R_i^j - \frac{1}{2}Rg_i^j = -(\rho u_i u^j - \lambda x_i x^j), \tag{2}$$

where u_i and x_i satisfy conditions

$$u^i u_i = -x^i x_i = -1, \tag{3}$$

and

$$u^i x_i = 0. \tag{4}$$

Here, ρ is the rest energy of the cloud of strings with massive particles attached to them. $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the density of tension that characterizes the strings. The unit space-like vector x^i represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector u^i describes the four-velocity vector of the matter satisfying the following conditions

$$g_{ij}u^i u^j = -1. \tag{5}$$

In the present scenario, the comoving coordinates are taken as

$$u^i = \left(0, 0, 0, \frac{1}{A}\right) \tag{6}$$

and choose x^i parallel to x-axis so that

$$x^i = \left(\frac{1}{A}, 0, 0, 0\right). \tag{7}$$

The Einstein's field equations (2) for the line-element (1) lead to the following system of equations:

$$G_1^1 \equiv \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC}$$

$$= \lambda A^2, \tag{8}$$

$$G_2^2 \equiv \left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{C_{44}}{C} - \frac{C_{11}}{C} = 0, \tag{9}$$

$$G_3^3 \equiv \left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} = 0, \tag{10}$$

$$G_4^4 \equiv -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC}$$

$$= \rho A^2, \tag{11}$$

$$G_4^1 \equiv \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \tag{12}$$

where the sub indices 1 and 4 in A, B, C and elsewhere denote differentiation with respect to x and t, respectively.

The velocity field u^i is irrotational. The scalar expansion θ, shear scalar σ^2, acceleration vector $\dot{u}_i$ and proper volume V^3 are respectively found to have the following expressions:

$$\theta = u^i_{;i} = \frac{1}{A}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right), \tag{13}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\theta^2 - \frac{1}{A^2}\left(\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA}\right), \tag{14}$$

$$\dot{u}_i = u_{i;j}u^j = \left(\frac{A_1}{A}, 0, 0, 0\right), \tag{15}$$

$$V^3 = \sqrt{-g} = A^2 BC, \tag{16}$$

where g is the determinant of the metric (1). Using the field equations and the relations (13) and (14) one obtains the Raychaudhuri's equation as

$$\dot{\theta} = \dot{u}^i_{;i} - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}\rho_p, \tag{17}$$

where dot denotes differentiation with respect to t and

$$R_{ij}u^i u^j = \frac{1}{2}\rho_p. \tag{18}$$

With the help of equations (1) - (7), the Bianchi identity $\left(T^{ij}_{;j}\right)$ reduced to two equations:

$$\rho_4 - \frac{A_4}{A}\lambda + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\rho = 0 \tag{19}$$

and

$$\lambda_1 - \frac{A_1}{A}\rho + \left(\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}\right)\lambda = 0. \tag{20}$$

Thus due to all the three (strong, weak and dominant) energy conditions, one finds $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of λ is unrestricted, it may take values positive, negative or zero as well.

3 Solutions of the Field Equations

As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the string cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non- linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit.

Here to get a determinate solution, let us assume that expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. This condition leads to

$$A = (BC)^n, \tag{21}$$

where n is a constant. Equations (9) and (10) lead to

$$\frac{B_{44}}{B} - \frac{B_{11}}{B} = \frac{C_{44}}{C} - \frac{C_{11}}{C}. \tag{22}$$

Using (21) in (12), yields

$$\frac{B_{41}}{B} + \frac{C_{41}}{C} - 2n\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\left(\frac{B_1}{B} + \frac{C_1}{C}\right) = 0. \tag{23}$$

To find out deterministic solutions, we consider the following three cases:

$$(i) B = f(x)g(t) \;\; \text{and} \;\; C = h(x)k(t),$$

$$(ii) B = f(x)g(t) \;\; \text{and} \;\; C = f(x)k(t),$$

$$(iii) B = f(x)g(t) \;\; \text{and} \;\; C = h(x)g(t).$$

The case (iii) is discussed by Kilinc and Yavuz (1996). We obtain a new class of solutions for other two cases (i) and (ii) and discuss their consequences separately below in this paper.

3.1 Case(i):

$B = f(x)g(t)$ and $C = h(x)k(t)$

In this case equation (23) reduces to

$$\frac{f_1/f}{h_1/h} = -\frac{(2n-1)(k_4/k) + 2n(g_4/g)}{(2n-1)(g_4/g) + 2n(k_4/k)} = K(\text{constant}), \tag{24}$$

which leads to

$$\frac{f_1}{f} = K\frac{h_1}{h} \tag{25}$$

and

$$\frac{k_4/k}{g_4/g} = \frac{K - 2nK - 2n}{2nK + 2n - 1} = a(\text{constant}). \tag{26}$$

From Eqs. (25) and (26), we obtain

$$f = \alpha h^K \tag{27}$$

and

$$k = \delta g^a, \tag{28}$$

where α and δ are integrating constants. Eq. (22) reduces to

$$\frac{g_{44}}{g} - \frac{k_{44}}{k} = \frac{f_{11}}{f} - \frac{h_{11}}{h} = N, \tag{29}$$

where N is a constant. Using the functional values of B and C in (22), we obtain

$$gg_{44} + ag_4^2 = -\frac{p^2}{(1+a)}g^2, \tag{30}$$

which leads to

$$g = \left(b_1 e^{pt} + b_2 e^{-pt}\right)^{\frac{1}{a+1}}, \tag{31}$$

where

$$p = \sqrt{\frac{N(a+1)}{(1-a)}}$$

and b_1, b_2 are constants of integration. Thus from Eq. (28) we get

$$k = \delta\left(b_1 e^{pt} + b_2 e^{-pt}\right)^{\frac{a}{a+1}}. \tag{32}$$

From Eqs. (25) and (29), we obtain

$$hh_{11} + Kh_1^2 = \frac{q^2}{(K+1)}h^2, \tag{33}$$

which leads to

$$h = \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{\frac{1}{K+1}}, \tag{34}$$

where

$$q = \sqrt{\frac{N(K+1)}{(K-1)}}$$

and c_1, c_2 are constants of integration. Hence from Eq. (27) we have

$$f = \alpha \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{\frac{K}{K+1}}. \tag{35}$$

It is worth mentioned here that equations (30) and (33) are fundamental basic differential equations for which we have reported new solutions given by equations (31) and (34).
Thus, we obtain

$$B = fg = \alpha \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{\frac{K}{K+1}} \left(b_1 e^{pt} + b_2 e^{-pt}\right)^{\frac{1}{a+1}}, \tag{36}$$

$$C = hk = \delta \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{\frac{1}{K+1}} \left(b_1 e^{pt} + b_2 e^{-pt}\right)^{\frac{a}{a+1}}. \tag{37}$$

Therefore

$$A = (BC)^n = (\alpha\delta)^n \left(c_1 e^{qx} + c_2 e^{-qx}\right)^n \left(b_1 e^{pt} + b_2 e^{-pt}\right)^n, \tag{38}$$

Hence the metric (1) takes the form

$$\begin{aligned} ds^2 = (\alpha\delta)^{2n} \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{2n} \left(b_1 e^{pt} + b_2 e^{-pt}\right)^{2n} (dx^2 - dt^2) + \\ \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{\frac{2}{K+1}} \left(b_1 e^{pt} + b_2 e^{-pt}\right)^{\frac{2}{a+1}} \Big[\alpha^2 \left(c_1 e^{qx} + c_2 e^{-qx}\right)^{\frac{2(K-1)}{K+1}} dy^2 \\ + \delta^2 \left(b_1 e^{pt} + b_2 e^{-pt}\right)^{\frac{2(a-1)}{(a+1)}} dz^2\Big]. \end{aligned} \tag{39}$$

The energy density (ρ), the string tension density (λ) and the particle density (ρ_p) for the model (46) are given by

$$\begin{aligned} \rho = \frac{1}{(\alpha\delta)^{2n}\phi(x)^{2n}\psi(t)^{2n}} \Bigg[-\frac{4q^2 c_1 c_2}{\phi(x)^2} - \frac{q^2\eta(x)^2}{\phi(x)^2} \left\{ \frac{K^2+K+1}{(K+1)^2} - n \right\} \\ + \frac{p^2\xi(t)^2}{\psi(t)^2} \left\{ \frac{a}{(a+1)^2} + n \right\} \Bigg], \end{aligned} \tag{40}$$

$$\begin{aligned} \lambda = \frac{1}{(\alpha\delta)^{2n}\phi(x)^{2n}\psi(t)^{2n}} \Bigg[\frac{4p^2 b_1 b_2}{\psi(t)^2} + \frac{p^2\xi(t)^2}{\psi(t)^2} \left\{ \frac{a^2+a+1}{(a+1)^2} - n \right\} \\ + \frac{q^2\eta(x)^2}{\phi(x)^2} \left\{ \frac{K}{(K+1)^2} + n \right\} \Bigg], \end{aligned} \tag{41}$$

$$\rho_p = \frac{1}{(\alpha\delta)^{2n}\phi(x)^{2n}\psi(t)^{2n}} \left[-\frac{4q^2 c_1 c_2}{\phi(x)^2} - \frac{4p^2 b_1 b_2}{\psi(t)^2} - \frac{q^2\eta(x)^2}{\phi(x)^2} + \frac{p^2\xi(t)^2}{\psi(t)^2} \right], \tag{42}$$

where

$$
\begin{aligned}
\phi(x) &= c_1 e^{qx} + c_2 e^{-qx}, \\
\psi(t) &= b_1 e^{pt} + b_2 e^{-pt}, \\
\eta(x) &= c_1 e^{qx} - c_2 e^{-qx}, \\
\xi(t) &= b_1 e^{pt} - b_2 e^{-pt}.
\end{aligned} \tag{43}
$$

The scalar of expansion (θ), shear tensor (σ), the acceleration vector ($\dot{u}_i$) and the proper volume (V^3) for the model (46) are obtained as

$$\theta = \frac{p(n+1)\xi(t)}{(\alpha\delta)^n \phi(x)^n \psi(t)^{n+1}}, \tag{44}$$

$$\sigma^2 = \frac{p^2 \xi(t)^2}{(\alpha\delta)^{2n} \phi(x)^{2n} \psi(t)^{2n+2}} \left[\frac{1}{3}(n+1)^2 - \left(n + \frac{a}{(a+1)^2} \right) \right], \tag{45}$$

$$\dot{u}_i = \left(\frac{nq\eta(x)}{\phi(x)}, 0, 0, 0 \right), \tag{46}$$

$$V^3 = \sqrt{-g} = (\alpha\delta)^{2n+1} \phi(x)^{2n+1} \psi(t)^{2n+1}. \tag{47}$$

From equations (44) and (45), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{n(a+1)^2 + a}{(a+1)^2 (n+1)^2} = (\text{constant}). \tag{48}$$

3.2 Case(ii):

$B = f(x)g(t)$ and $C = f(x)k(t)$

In this case equation (23) reduces to

$$(4n-1)\frac{f_1}{f}\left(\frac{g_4}{g} + \frac{k_4}{k} \right) = 0. \tag{49}$$

The equation (49) leads to three cases:

$$
\begin{aligned}
&(a) \quad n = \frac{1}{4}, \\
&(b) \quad \frac{f_1}{f} = 0, \\
&(c) \quad \frac{g_4}{g} + \frac{k_4}{k} = 0.
\end{aligned}
$$

The case (a) reduces the number of equation to four but, with five unknowns which requires additional assumption for a viable solution. In the case (b), the model turns to be a particular case to the Bianchi type-I model. Therefore we

consider the case (c) only.

Using condition (c) in equation (22) leads to

$$\frac{g_{44}}{g} = \frac{k_{44}}{k}. \tag{50}$$

By using condition (c) in (50), we get

$$g = e^{\ell t}, \quad k = e^{-\ell t}, \tag{51}$$

where ℓ is constant.

From equations (9) or (10) and (51), we have

$$ff_{11} - \frac{2n}{2n+1}f_1^2 - \frac{\ell^2}{2n+1}f^2 = 0. \tag{52}$$

Solving (52), we obtain

$$f = \left(d_1 e^{rx} + d_2 e^{-rx}\right)^{2n+1}, \tag{53}$$

where d_1 and d_2 are constants of integration and

$$r = \frac{\ell}{2n+1}.$$

It is important to mention here that (52) is the basic equation for which new solution is obtained as given by (53).

Hence, we obtain

$$B = fg = \left(d_1 e^{rx} + d_2 e^{-rx}\right)^{2n+1} e^{\ell t} \tag{54}$$

and

$$C = fk = \left(d_1 e^{rx} + d_2 e^{-rx}\right)^{2n+1} e^{-\ell t}. \tag{55}$$

Therefore

$$A = (BC)^n = \left(d_1 e^{rx} + d_2 e^{-rx}\right)^{2n(2n+1)}. \tag{56}$$

In this case the metric (1) reduces to the form

$$ds^2 = (d_1 e^{rx} + d_2 e^{-rx})^{4n(2n+1)}(dx^2 - dt^2) + (d_1 e^{rx} + d_2 e^{-rx})^2 (e^{2\ell t} dy^2 + e^{-2\ell t} dz^2). \tag{57}$$

In this case the physical parameters ρ, λ, ρ_p and kinematical parameters θ, σ, $\dot{u}_i$ and V^3 for the model (57) are given by

$$\rho = \frac{1}{\mu(x)^{4n(2n+1)}}\left[-\frac{8(2n+1)d_1 d_2 r^2}{\mu(x)^2} + \frac{(2n+1)^2(4n-3)r^2\nu(x)^2}{\mu(x)} + \ell^2\right], \tag{58}$$

$$\lambda = \frac{1}{\mu(x)^{4n(2n+1)}}\left[-\frac{(4n+1)(2n+1)^2 r^2 \nu(x)^2}{\mu(x)^2} + \ell^2\right], \tag{59}$$

$$\rho_p = \frac{2}{\mu(x)^{4n(2n+1)}}\left[-\frac{4(2n+1)d_1 d_2 r^2}{\mu(x)^2} + \frac{(2n+1)^2(4n-1)r^2\nu(x)^2}{\mu(x)^2}\right], \tag{60}$$

$$\theta = 0, \tag{61}$$

$$\sigma^2 = \frac{\ell^2}{\mu(x)^{4n(2n+1)}}, \tag{62}$$

$$\dot{u}_i = \Big(\frac{2n(2n+1)r\nu(x)}{\mu(x)}, 0, 0, 0\Big), \tag{63}$$

$$V^3 = \sqrt{-g} = \mu(x)^{2(2n+1)^2}, \tag{64}$$

where

$$\mu(x) = d_1 e^{rx} + d_2 e^{-rx},$$
$$\nu(x) = d_1 e^{rx} - d_2 e^{-rx}. \tag{65}$$

4 Concluding Remarks

In the study, we have presented a new class of exact solutions of Einstein's field equations for inhomogeneous cylindrically symmetric space-time with string sources which are different from the other author's solutions. In these solutions all physical quantities depend on at most one space coordinate and the time.

In case (i), the models (39) represents expanding, shearing and non-rotating universe. The expansion in the model increases as time increases when $n < 0$ but the expansion in the model decreases as time increases when $n > 0$. The spatial volume increases as time increases. If we set the suitable values of constants, we find that energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. We observe that $\frac{\sigma}{\theta}$ is constant throughout. Shear (σ) vanishes when $p = 0$. Thus the model isotropizes when $p = 0$. The acceleration vector $\dot{u}$ is zero for $n = 0$. In this model all the physical and kinematic parameters vanish for $p = 0$, $q = 0$. The model (39) represents a realistic model. In this solution all physical and kinematical quantities depend on at most one space coordinate and the time.

In case (ii), the expansion θ, in model (57), is zero. With the help of physical and kinematical parameters, we can determine some physical and geometric features of the model. All kinematical quantities are independent of T. In general, the model represents non-expanding, non-rotating and shearing universe. The spatial volume V increases as distance increases. The shear (σ) vanishes when $\ell = 0$. The acceleration vector $\dot{u}$ is zero for $n = 0$, $n = -\frac{1}{2}$. Choosing suitable values for constants n and L, we find that energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. The solutions identically satisfy the Bianchi identities given by (19) and (20). In this solution all physical and kinematical quantities depend on at most one space coordinate.

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